

Finite temperature effects in trapped Fermi gases with population imbalance

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We study the finite temperature T behavior of trapped Fermi gases as they undergo BCS-Bose Einstein condensation (BEC) crossover, in the presence of a population imbalance. Our results, in qualitative agreement with recent experiments, show how the superfluid phase transition is directly reflected in the particle density profiles. We demonstrate that at $T \neq 0$ and in the near-BEC and unitary regimes, the polarization is excluded from the superfluid core. Nevertheless a substantial polarization fraction is carried by a normal region of the trap having strong pair correlations, which we associate with noncondensed pairs or the “pseudogap phase”.

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Recent work [1, 2, 3] on trapped atomic Fermi gases with population imbalance has become particularly exciting. With the application of a magnetic field, these systems exhibit a continuous evolution [4, 5, 6] from BCS to Bose-Einstein condensation (BEC). Not only are these gases possible prototypes for condensed matter systems [7, 8] in the presence of a magnetic field - Zeeman coupling, but they may also be prototypes for particle and nuclear physics systems [9, 10]. These pioneering experiments have been done so far by two experimental groups [1, 2]. And what differences are present appear to lie more in the interpretation than in the actual data.

There are a number of key experimental observations which we now list. (i) Both groups have observed that the trap profiles are characterized by a central core of (at most) weakly polarized superfluid, surrounded by a *normal region where the bulk of the polarization is contained*. (ii) The normal region appears to consist of overlapping clouds of both spin states (“normal mixture”), followed at the edge of the cloud by a region consisting only of the majority component. There is not complete agreement [1, 2, 3] on whether the normal-superfluid boundary is sharp which would correspond to some form of phase separation.

These population imbalance experiments have been done [1, 3] in conjunction with other measurements (vortex excitations and magnetic field sweeps) which establish the presence and the location for superfluid condensation. (iii) Even more recently [3] it has been demonstrated that the superfluid phase transition at T_c can be directly reflected in changes in the shape of the clouds. (iv) Important for the present purposes is the fact that [3] there are strong interaction effects within the normal region of the cloud.

The goal of the present paper is to address the four points [(i)-(iv)] listed above through a finite temperature theory of BCS-BEC crossover in the presence of population imbalance within a trap. A related study of the homogeneous system was presented earlier [11]. What is unique to our work is the capability of separating in a natural way the condensed from noncondensed pair contributions to the trap profile. The difficulty in making this separation lies in the fact that (except at $T = 0$) the presence of a fermionic excitation gap, is *not* a signature of phase coherent superconductivity. We also present calculations of T_c in a trap and show how the general shape of the profile changes below and above T_c , unlike what is found

experimentally and theoretically [12] in the absence of polarization. In a related fashion, we examine the noncondensed pair states in the trap and determine to what extent they differ from a free gas mixture of the two spin states.

Our principal findings are at general $T \neq 0$ and for the unitary and near-BEC regimes, (a) the superfluid core seems to be robustly maintained at nearly zero polarization. (b) The mixed normal region, carries a significant fraction of the polarization within a non-superfluid state having strong pair correlations. Indeed, experiments suggest [3] that “even in the normal state, strong interactions significantly deform the density profile of the majority spin component”. Here we interpret these correlations as noncondensed pairs which have no counterpart at $T = 0$ and which are associated with an excitation gap (“pseudogap”) in the fermionic spectrum. Finally, (c) in the course of making contact with points (i)-(iv) listed above we show good qualitative agreement with experiment.

Because we restrict our attention to condensates (and their pseudogap phase counterparts) with zero momentum ($q_0 = 0$) pairing, we do not explore those regimes of the phase diagram corresponding to the lowest temperatures, and highest polarizations. Recent very nice theoretical work based on the Bogoliubov-de Gennes (BdG) approach [13, 14], has shown that in the *ground* state at unitarity the $q_0 \neq 0$, Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [15] must be incorporated. Importantly, the polarization in this state at $T = 0$ appears at the edge but *within the condensate* [14]. Fortunately, the present work provides a good indication of where the FFLO phase will enter, since it occurs when the $q_0 = 0$ phase is found to be unstable [14, 16]. With the finite T required by experiments, it is quite possible that the FFLO phase presents itself, primarily in the form of noncondensed pairs, which lie on the perimeter of the condensate.

The value of the present work is derived from the fact that a central theme in the experimental literature involves distinguishing the condensate from the normal regions of the trap. Whether there is phase separation or not [1, 2] the precise nature of the normal (N) and superfluid (S) phases are all of great interest. In this way one needs a theory which distinguishes N from S at finite temperatures, where the excitation gap is no longer a signature of superfluidity. Previous theoretical approaches, based on the BdG [13, 14] and local density approximation (LDA) [17, 18, 19, 20] schemes have empha-

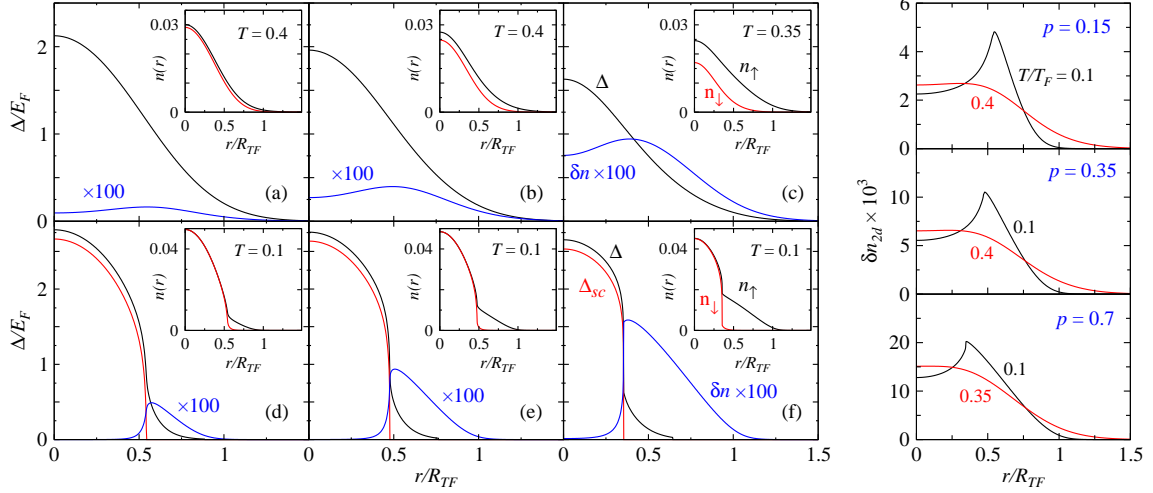


Figure 1: (Color online) Spatial distribution of the excitation gap $\Delta(r)$ (black) and order parameter $\Delta_{sc}(r)$ (red, main figures) and density $n_\sigma(r)$ (insets) at $1/k_F a = 1.5$ for the majority (black) and minority (red) fermions at different polarizations ($p = 0.15, 0.35$, and 0.7 from left to right) above (upper row) and below (lower row) T_c . Here $T_c/T_F \approx 0.36, 0.35$, and 0.31 , respectively. The density difference $\delta n(r)$ is shown in blue in the main figures, sharing the same vertical axis as $\Delta(r)$. The temperatures for the upper row are $T/T_F = 0.4, 0.4$, and 0.35 , respectively, and for the lower row $T/T_F = 0.1$. Shown on the far right is the difference in column density, $\delta n_{2d}(r)$, for the three polarizations above (red) and below (black) T_c . Here $E_F = \hbar^2 k_F^2 / 2m = k_B T_F$ is given by the Fermi energy for an unpolarized, noninteracting Fermi gas with the same total number N at $T = 0$, and $R_{TF} \equiv \sqrt{2E_F / m\omega^2}$ is the Thomas-Fermi radius. The units for n and δn_{2d} are k_F^3 and k_F^2 , respectively.

sized $T = 0$, albeit without reaching any clear consensus. The inclusion of finite T for the LDA case has been introduced within the same formalism we use here [8], but without separating the condensed and noncondensed pair contributions. In addition the application of BdG to $T \neq 0$ is viewed as problematic because it does not incorporate noncondensed pairs [5, 21, 22]. At the same time, it should be stressed that this BdG approach [13, 14] is most likely the appropriate way to get a full picture of the $T = 0$ phase.

The formalism used in this paper was outlined earlier [11]. Here we incorporate trap effects by use of the LDA. We adopt a one-channel approach since the ^6Li resonances studied thus far are broad and consider a Fermi gas of two spin species with kinetic energy $\epsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m$ subject to an attractive contact potential ($U < 0$) between the different spin states. We define $\delta n = n_\uparrow - n_\downarrow > 0$, where $n = n_\uparrow + n_\downarrow$ is the total atomic density. Importantly, we include [23] noncondensed pairs at general T . The T -matrix or noncondensed pair propagator, is $t(Q) = U / [1 + U\chi(Q)]$, where $\chi(Q)$ is the pair susceptibility discussed earlier which depends self consistently on the fermionic excitation gap Δ . The presence of pairing correlations means that Δ^2 contains two additive contributions from the condensed (Δ_{sc}^2) and noncondensed pairs (Δ_{pg}^2). In the superfluid phase, we have $1 + U\chi(0) = 0$, equivalent to $\mu_{pair} = 0$, the BEC condition of the pairs. As a consequence, the equations become simpler below T_c and we may expand the T -matrix to arrive at a characteristic frequency $\Omega_{\mathbf{q}}$ which characterizes the dispersion of the noncondensed pairs.

We now summarize the self-consistent equations [12, 23], in the presence of a spherical trap, treated at the level of LDA with trap potential $V(r) = \frac{1}{2}m\omega^2 r^2$. T_c is defined as the

highest temperature at which the self-consistent equations are satisfied precisely at the center. At a temperature $T < T_c$ the superfluid region extends to a finite radius R_{sc} . The particles outside this radius are in a normal state, with or without a pseudogap.

The generalized local gap equation is given by

$$\frac{m}{4\pi a} = \sum_{\mathbf{k}} \left[\frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1 - 2\tilde{f}(E_{\mathbf{k}})}{E_{\mathbf{k}}} \right] + Z\mu_{pair}, \quad (1)$$

where $\mu_{pair}(r) = 0$ in the superfluid region $r \leq R_{sc}$, and must be solved for self-consistently at larger radii. The quantity Z is the inverse residue of the T -matrix [11]. For convenience we write $\tilde{f}(x) \equiv [f(x+h) + f(x-h)]/2$ where $f(x)$ is the Fermi distribution function. Here we set $\hbar = 1$, and use $m/4\pi a = 1/U + \sum_{\mathbf{k}} (2\epsilon_{\mathbf{k}})^{-1}$ to regularize the contact potential, where a is the two-body s -wave scattering length. The dispersion is given by $E_{\mathbf{k}} = \sqrt{[\epsilon_{\mathbf{k}} - \mu(r)]^2 + \Delta^2}$, with $\mu(r) = (\mu_\uparrow + \mu_\downarrow)/2 - V(r)$. We also define the r -independent parameter $h = (\mu_\uparrow - \mu_\downarrow)/2$. Since $\delta n \geq 0$, we always have $h > 0$. More generally, μ_σ is the chemical potential for spin σ at the trap center.

The local pseudogap contribution (present only at $T \neq 0$) to $\Delta^2(T) = \Delta_{sc}^2(T) + \Delta_{pg}^2(T)$ is given by

$$\Delta_{pg}^2 = \frac{1}{Z} \sum_{\mathbf{q}} b(\Omega_{\mathbf{q}} - \mu_{pair}). \quad (2)$$

where $b(x)$ is the usual Bose distribution function. The density of particles at radius r can be written as

$$n_\sigma(r) = \sum_{\mathbf{k}} [u_{\mathbf{k}}^2 f(E_{\mathbf{k}\sigma}) + v_{\mathbf{k}}^2 f(-E_{\mathbf{k}\bar{\sigma}})], \quad (3)$$

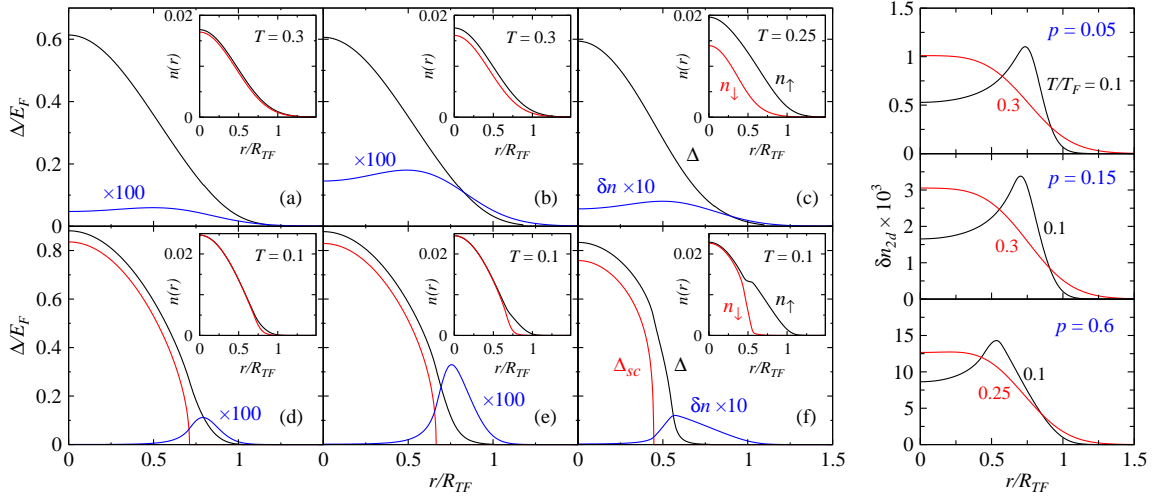


Figure 2: Same as Fig. 1 but at unitarity. From left to right, $p = 0.05, 0.15$, and 0.6 , and $T_c/T_F \approx 0.27, 0.27$, and 0.23 , respectively. For the upper row, $T/T_F = 0.3, 0.3$, and 0.25 , respectively, and for the lower row $T/T_F = 0.1$. Shown on the far right is $\delta n_{2d}(r)$.

which depends on the coherence factors $u_{\mathbf{k}}, v_{\mathbf{k}}^2 = (1 \pm \xi_{\mathbf{k}}/E_{\mathbf{k}})/2$ with $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu(r)$, and $E_{\mathbf{k}\uparrow} = -h + E_{\mathbf{k}}$, and $E_{\mathbf{k}\downarrow} = h + E_{\mathbf{k}}$. The total number of particles and the polarization are respectively given by

$$N_{\sigma}(r) = \int d^3r n_{\sigma}(r), \quad N = N_{\uparrow} + N_{\downarrow}, \quad (4)$$

$$p = (N_{\uparrow} - N_{\downarrow})/N. \quad (5)$$

Our calculations proceed by numerically solving the self-consistent equations. Here we use the N and p as input; these are the control parameters in experiment.

Figure 1 shows the behavior of the various gap parameters and the majority and minority spin components as a function of radius in the trap, for the case of a near-BEC system with $1/k_F a = 1.5$. The upper panels are for the normal phase and the lower panels are in the superfluid state. We present results for three different polarizations and focus first on the lower panels where there are two distinct components to the gap Δ_{sc} and Δ_{pg} . The two gap functions, Δ_{sc} and the total gap Δ , are plotted vs r along with the difference density for up and down spins, or alternatively, the polarization. We overlay these plots into order to show clearly what are the contributions to the polarization from the condensate (I), where $\Delta_{sc} \neq 0$, the correlated, but normal mixed region (II), where $\Delta_{sc} = 0$, but $\Delta \neq 0$, and non-interacting Fermi gas(s) regime (III), where $\Delta = 0$.

It can be seen that there is very little polarization present in the condensate (I) which appears below T_c , as has been inferred experimentally [1, 2, 3]. Rather the bulk of the polarization is present in the correlated, but normal region (II) in which there is a finite excitation gap Δ , but vanishing Δ_{sc} . In region III at even larger radii, Δ is essentially zero and region III is predominantly composed of the majority spin component. In this regime, one expects the cloud wing-shape to be that of a non-interacting Fermi gas, and this provides the basis for a reasonable thermometry [3]. As T is lowered the

noncondensed pairs in Region II will be converted into the condensate, thereby merging Regions I and II.

Because we have not yet incorporated the $q_0 \neq 0$ correlations of the FFLO state, in Figs. 1 and 2 the largest of 3 values of p used is associated with an instability at the very edge of the minority cloud. Nevertheless, since n_{\downarrow} essentially vanishes there, this is expected to have very little qualitative effect on our results.

The insets in the lower panel show the density profiles for the majority (in black) and minority (in red) component. Qualitatively similar to what has been observed experimentally [3], a small “kink” in the majority is present at the radius at which the condensate ends. The minority component contains essentially only a condensate central peak with a very weak bi-modal structure. The upper panel shows the behavior in the normal state, where the condensate $\Delta_{sc} = 0$. Nevertheless, it can be seen that an excitation gap Δ is present throughout the cloud. In this way the particle profiles do not correspond to those of a non-interacting gas, and the polarization is rather evenly distributed at all radii in the cloud. It may also be seen from these insets that as the system varies from above to below T_c , the profile of the minority component contracts into the center of the trap, as observed [3].

We present comparable figures for the unitary case in Fig. 2. Most of the observations made above for the near-BEC case obtain at unitarity as well. Here, one can see from the insets, however, that the kink in the majority profile is less apparent. Finally, we turn to Fig. 3 where the counterpart plots are presented for the BEC regime with $1/k_F a = 3.0$. From the right column, it may be seen that polarization penetrates down to the trap center, when the overall polarization p is high. This is in agreement with the expectations from the homogeneous case [11].

It should be stressed that our calculations indicate that dramatic changes in the shape of the density profiles do not occur until T is substantially lower than T_c . This may explain why the experimentally observed T_c values are less [3] than

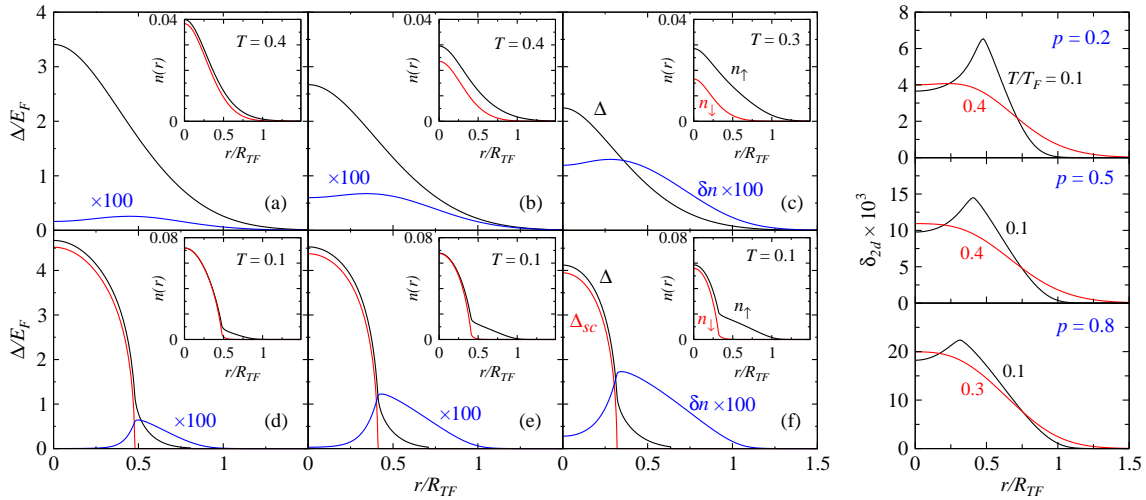


Figure 3: Same as Fig. 1 but for $1/k_F a = 3$. From left to right, $p = 0.2, 0.5$, and 0.8 , and $T_c/T_F \approx 0.39, 0.36$, and 0.28 , respectively. For the upper row, $T/T_F = 0.4, 0.4$, and 0.3 , respectively, and for the lower row $T/T_F = 0.1$. Shown on the far right is $\delta n_{2d}(r)$.

those we compute. Also important is the fact that the mixed normal phase we find here (Region II) is not related to that introduced at $T = 0$ in other theoretical work [18, 19]. The noncondensed pair contribution we consider has no counterpart at $T = 0$. Moreover, there is no evidence from $T = 0$ BdG investigations [13, 14] for such a mixed normal phase, although whether it is consistent with the FFLO state in a trap bears further investigation.

Essentially all the qualitative observations reported in this paper correspond to their counterparts in Ref. [3] with one exception. In Ref. [3], it is claimed that only regions I and III are present in the near-BEC regime, whereas, we find all three regions appear, just as in the unitary case. Region II corresponds quite naturally to the presence of noncondensed pairs, expected at finite T in the near-BEC regime. At a more quan-

titative level, our results at unitarity may change somewhat when we include FFLO condensate contributions, and associated higher values of p . While we do not find evidence for sharp phase separation as reported in Ref. [2], after column integration of Figs. 1-3, a double peaked structure emerges for the difference profile δn_{2d} at low T , as has been claimed experimentally [1, 2]. Importantly, as stressed in Ref. [3], and, as is consistent with our longstanding viewpoint [6], these studies show that a significant fraction of the normal region in the trap contains strong interactions between the two spin states which we associate with the presence of noncondensed pairs and related fermionic excitation (pseudo)gap.

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- [1] M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, *Science* **311**, 492 (2006).
 - [2] G. B. Partridge, W. Li, R. I. Kamar, Y. A. Liao, and R. G. Hulet, *Science* **311**, 503 (2006).
 - [3] M. W. Zwierlein, H. Schunck, A. Schirotzek, and W. Ketterle, *cond-mat/0605258* (2006).
 - [4] A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter* (Springer-Verlag, Berlin, 1980), pp. 13–27.
 - [5] Q. J. Chen, I. Kosztin, B. Jankó, and K. Levin, *Phys. Rev. Lett.* **81**, 4708 (1998).
 - [6] Q. J. Chen, J. Stajic, S. N. Tan, and K. Levin, *Phys. Rep.* **412**, 1 (2005).
 - [7] D. Sheehy and L. Radzihovsky, *Phys. Rev. Lett.* **96**, 060401 (2006).
 - [8] W. Yi and L. M. Duan, *Phys. Rev. A* **73**, 031604(R) (2006).
 - [9] W. V. Liu and F. Wilczek, *Phys. Rev. Lett.* **90**, 047002 (2003).
 - [10] M. M. Forbes, E. Gubankova, W. V. Liu, and F. Wilczek, *Phys. Rev. Lett.* **94**, 017001 (2005).
 - [11] C.-C. Chien, Q. J. Chen, Y. He, and K. Levin, *cond-mat/0605039* (2006).
 - [12] J. Stajic, Q. J. Chen, and K. Levin, *Phys. Rev. Lett.* **94**, 060401 (2005).
 - [13] J. Kinnunen, L. M. Jensen, and P. Torma, *Phys. Rev. Lett.* **96**, 110403 (2006).
 - [14] K. Machida, T. Mizushima, and M. Ichioka, *cond-mat/0604339* (2006).
 - [15] P. Fulde and R. A. Ferrell, *Phys. Rev.* **135**, A550 (1964); A. I. Larkin and Y. N. Ovchinnikov, *Zh. Exp. Teor. Fiz.* **47**, 1136 (1964) [*Sov. Phys. JETP* **20**, 762 (1965)].
 - [16] L. Y. He, M. Jin, and P. F. Zhuang, preprint.
 - [17] C. H. Pao, S. T. Wu, and S. K. Yip, *Phys. Rev. B* **73**, 132506 (2006).
 - [18] T. N. De Silva and E. J. Mueller, *cond-mat/0601314* (2006).
 - [19] M. Haque and H. T. C. Stoof, *cond-mat/0601321* (2006).
 - [20] P. Pieri and G. C. Strinati, *cond-mat/0512354* (2005).
 - [21] I. Kosztin, Q. J. Chen, B. Jankó, and K. Levin, *Phys. Rev. B* **58**, R5936 (1998).
 - [22] P. Pieri and G. C. Strinati, *Phys. Rev. Lett.* **91**, 030401 (2003).
 - [23] J. Stajic, J. N. Milstein, Q. J. Chen, M. L. Chiofalo, M. J. Holland, and K. Levin, *Phys. Rev. A* **69**, 063610 (2004).